7th Grade Advanced Integrated Math

UNIT 1: Real Numbers

ESSENTIAL QUESTION	BIG IDEAS
How can visuals and models be used to represent rational and irrational number operations in our world?	Students explore the connections between the four operations. Students will use models to represent operations with rational and irrational numbers. Students will solve and interpret real-world and mathematical problems involving the four operations with rational numbers and irrational numbers.

GUIDING QUESTIONS

Content and Process

- How can a numberline be used to represent addition and subtraction? **7.NS.1**
- How are models used to prove that opposites combine to 0? **7.NS.1a**
- How does understanding absolute value help when adding and subtracting rational numbers? **7.NS.1b, 7.NS.1d**
- How can the additive inverse be used to model the subtraction of rational numbers? **7.NS.1c**
- How can the properties of operations be used as a strategy to add, subtract, multiply, and divide rational numbers? **7.NS.1e, 7.NS.2c**
- How is multiplying and dividing negative numbers related to multiplying and dividing positive numbers? **7.NS.2**
- What patterns are present when multiplying and dividing signed numbers? **7.NS.2a, 7.NS.2b**
- What methods can be used to convert between fractions and decimals, including repeating decimals? **7.NS.2d, 8.NS.1**
- How do the characteristics of a decimal determine if a number is rational or irrational? **7.NS.2d, 8.NS.1**
- How can operations with rational numbers be used to solve real-world problems? **7.NS.3**

Reflective

- What surprised you when operating with rational and irrational numbers?
- Using a model, how would you show a friend how to add and subtract integers?
- Why is it important for me to understand integer operations in our world?

FOCUS STANDARDS

Standards of Mathematical Practice

MP. 4 Model with mathematics
MP. 6 Attend to precision

Content Standards
7.NS.1. Represent addition and subtraction on a horizontal or vertical number line diagram.

- **7.NS.1a.** Describe situations in which opposite quantities combine to make 0. Show that a number and its opposite have a sum of 0 (are additive inverses). *For example, show zero-pairs with two-color counters.*
- **7.NS.1b.** Show $p + q$ as the number located a distance $|q|$ from *p*, in the positive or negative direction depending on whether *q* is positive or negative.
- **7.NS.1c.** Model subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$.
- **7.NS.1d.** Model subtraction as the distance between two rational numbers on the number line where the distance is the absolute value of their difference.
- **7.NS.1e.** Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2. Apply and extend previous understandings of multiplication and division of positive rational numbers to multiply and divide all rational numbers.

- **7.NS.2a.** Describe how multiplication is extended from positive rational numbers to all rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers.
- **7.NS.2b.** Explain that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. Leading to situations such that if *p* and *q* are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$.
- **7.NS.2c.** Apply properties of operations as strategies to multiply and divide rational numbers.
- **7.NS.2d.** Convert a rational number in the form of a fraction to its decimal equivalent using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3. Solve and interpret real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)
8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

UNIT 2: Expressions, Equations, and Inequalities

ESSENTIAL QUESTION	BIG IDEAS
How can algebraic concepts be used to represent real-world situations?	Students will apply and make use of percentages in real-world situations. Students will use all four operations to generate equivalent expressions. Students will write and apply equations or inequalities to real-world situations, to find solutions.

GUIDING QUESTIONS

Content and Process

- How can the properties of operations be used to add, subtract, factor, and expand linear expressions? **7.EE.1**
- How can various forms of expressions show how quantities are related? **7.EE.2**
- How can rational numbers in equivalent forms be used to solve multi-step real-life problems? **7.EE.3**
- How can estimation be used and mental computation to assess the reasonableness of answers? **7.EE.3**
- How can two-step equations and inequalities be used to make sense of real world situations? **7.EE.4**
- How can rational numbers in word problems be used to construct equations and solve for unknowns? **7.EE.4a**
- How can rational numbers in word problems be used to construct inequalities and solve for unknowns? **7.EE.4b**
- How can the solution set of an inequality be graphed on a number line? **7.EE.4b**

Reflective

- What real-world scenario could be used to represent the equation 2x + 3 = 13?
- What are two scenarios where you would use variables in your life?
- How could you explain to a friend how to graph y < 2x+1?

FOCUS STANDARDS

Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.

Content Standards

7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Note: factoring is limited to integer coefficients. *For example: apply the distributive property to the expression*
$24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$.
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by 5%" is the same as "multiply by 1.05."*
7.EE.3. Solve multi-step real-life and mathematical problems with rational numbers. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional*
$\frac{1}{10}$ *of her salary an hour, or $2.50, for a new salary of $27.50.*
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct two-step equations and inequalities to solve problems by reasoning about the quantities.

- **7.EE.4a.** Use variables to represent quantities in a real-world or mathematical problem, and construct two-step equations and inequalities to solve problems by reasoning about the quantities.

- **7.EE.4b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$ where p, q, and r are specific rational numbers and $p > 0$. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

7.RP.2c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t= pn.*

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

UNIT 3: Proportional Relationships

ESSENTIAL QUESTION	BIG IDEAS
How can proportional reasoning be applied to everyday life?	Students will visually represent proportional relationships with rational numbers and use them to solve real-world and mathematical problems. Students will use proportions to solve various percent problems. Students will use proportional reasoning to reproduce scale drawings of geometric figures.

GUIDING QUESTIONS

Content and Process

- How are the sides and area affected when scaling geometric figures? **7.G.1**
- How can scale drawings be used to reproduce figures at different sizes? **7.G.1**
- What strategies can be used to compute unit rates from a ratio of fractions? **7.RP.1**
- Why are two quantities considered proportional? **7.RP.2, 7RP.2a**
- How does the correlation between values in a table or points on a graph and their unit rate determine proportionality? **7. RP.2b**
- How can equations represent a proportional relationship? **7.RP.2**, **7.RP.2c**
- What do the points on a graph of a proportional relationship represent in terms of the situation? **7.RP.2d**
- How can proportions be used to solve multi-step ratio and percent problems? **7.RP.3**

Reflective

- How can I find the constant of proportionality in a table, graph, or equation?
- What proportional relationships in the world can you think of?
- For you, what is more useful for seeing proportional relationships, a table, graph or equation? Why?
- What coupon would you rather have, 20% off or $20 off?

FOCUS STANDARDS

Standards of Mathematical Practice
MP.4 Model with mathematics.
MP.7 Look for and make use of structure.

Content Standards

7.G.1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the* ***complex fraction*** $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour (interpreting a complex fraction as division of fractions), equivalently 2 miles per hour.*

7.RP.2. Recognize and represent proportional relationships between quantities:

- **7.RP.2.a.** Determine whether two quantities are in a proportional relationship*, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.*
- **7.RP.2b.** Analyze a table or graph and recognize that, in a proportional relationship, every pair of numbers has the same unit rate (referred to as the "m").
- **7.RP.2c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.
- **7.RP.2d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

UNIT 4: Exploring Roots

ESSENTIAL QUESTION	BIG IDEAS
How is the Pythagorean Theorem useful in real-life problem solving?	After discovering the formula for the Pythagorean Theorem, students will be able to explain why it works. Students will apply the Pythagorean Theorem to determine unknown side lengths on a coordinate plane and in real-world problems. Students can compare, locate, estimate, and evaluate the values of square and cubic roots.

GUIDING QUESTIONS

Content and Process Objectives

- Why does the Pythagorean Theorem formula work for all right triangles? **8.G.7**
- Given two sides of a right triangle in a two-dimensional figure, how can the Pythagorean Theorem be used to find the unknown side length? **8.G.8**
- How can the Pythagorean Theorem be used to find the distance between two points on a coordinate plane? **8.G.9**
- How can rational numbers be used to estimate the value of an irrational number and its location on a number line? **8.NS.2**
- How are the square or cubic roots used to find the solution to an equation? **8.EE.1**

Reflective

- How would you use a model or visual representation to demonstrate your understanding of the Pythagorean Theorem?
- What happens if $a^2 + b^2 > c^2$ or if $a^2 + b^2 < c^2$?
- How can you determine the distance between the two points (a,b) and (c,d)?
- How do you decide where $\sqrt{68}$ falls on the number line?

FOCUS STANDARDS

Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others.
MP.8 Look for and express regularity in repeated reasoning.

Content Standards

8.G.7 Explain a proof of the Pythagorean Theorem and its converse.

8.G.8 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world an mathematical problems in two and three dimensions. *For example: Finding the slant height of pyramids and cones.*

8.G.9 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions *(e.g.* π^2 *). For example, for the approximation of* $\sqrt{68}$ *, show that* $\sqrt{68}$ *is between 8 and 9 and closer to 8.*

8.EE.1 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 =$, where *p* is a positive rational number. Evaluate square roots of whole number perfect squares with solution between 0 and 15 and cube roots of whole number perfect cubes with solutions between 0 and 5. Know tha $\sqrt{2}$ is irrational.

UNIT 5: Geometric Figures

ESSENTIAL QUESTION	BIG IDEAS

How can understanding geometric figures help find answers to real-world problems?	Students will compose and decompose three-dimensional shapes to understa the connections between a two-dimensional and three-dimensional world. Students will be able to discover the connections between the arc length, circumference, area of sectors, and area in relation to pi. Students will use observations of three-dimensional shapes to derive formu to calculate the surface area and volume of prisms, pyramids, cones, spher and cylinders.

GUIDING QUESTIONS

Content and Process Objectives

- How does scaling a figure affect its volume? **7.G.1**
- When a rectangle or triangle is rotated around an edge, what 3-dimensional figure is formed? **7.G.2**
- Which shapes are created from various slices of three-dimensional figures? **7.G.3**
- How are diameter and circumference related to the area of a circle? **7.G.4**
- How can formulas be used to solve real-world problems involving circles? **7.G.4, 7.G.6**
- What are the relationships between various three-dimensional geometric shapes? **7.G.5, 8.G.11**
- Given two sides of a right triangle in a three-dimensional figure, how can the Pythagorean Theorem used to find the unknown side length? **8.G.8**
- How can the formula for the volume and surface area of prisms, pyramids, cones, spheres and cylinders be derived? **7.G.5a, 7.G.5b, 8.G.11.a, 8.G.11.b**
- How can arc length, area, area of sectors, surface area, and volume be applied to solve real-world problems? **7.G.6, 8.G.12**
- How can formulas and informal reasoning be used to find arc length, areas of sectors, surface areas and volumes of pyramids, cones, and spheres? **8.G.10**

Reflective

- Why are certain shapes better for minimizing surface area and maximizing volume?
- How would you explain to a friend how to calculate the perimeter and area of a quarter circle?
- What is the connection between the volume of cylinders, cones, and spheres that all have the same radius?

- What is the connection between how scale factor affects two-dimensional and three-dimensional figures?

FOCUS STANDARDS

Standards of Mathematical Practice

MP.1 Make sense and persevere in solving problems.

MP.5 Use appropriate tools strategically.

Content Standards

7.G.1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.2. Identify three-dimensional objects generated by rotating a two-dimensional (rectangular or triangula object around one edge.

7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right cylinder.

7.G.4. Use the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5. Investigate the relationship between three-dimensional geometric shapes;

- **7.G.5a.** Generalize the volume formula for prisms and cylinders (*V= bh* where *B* is the area of the ba and *h* is the height).
- **7.G.5b.** Generalize the surface area formula for prisms and cylinders (*SA = 2B + Ph* where *B* is the area of the base, *P* is the perimeter of the base, and *h* is the height (in the case of a cylinder, perime is replaced by circumference)).

7.G.6. Solve real-world and mathematical problems involving area of two-dimensional objects and volume and surface area of three-dimensional objects including cylinders and right prisms. (Solutions should **not** require students to take square roots or cube roots. *For example, given the volume of a cylinder and the ar of the base, students would identify the height.*)

8.G.8. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world a mathematical problems in two and three dimensions. *For example: Finding the slant height of pyramids and cones.*

8.G.10. Use the formulas or informal reasoning to find the arc length, areas of sectors, surface areas and volumes of pyramids, cones, and spheres. *For example, given a circle with a 60° central angle, students identify the arc length as*

$\frac{1}{6}$ *of the total circumference (* $\frac{1}{6} = \frac{60}{360}$ *).*

8.G.11. Investigate the relationship between the formulas of three dimensional geometric shapes;

- **8.G.11a.** Generalize the volume formula for pyramids and cones (*V = ⅓ B h*).
- **8.G.11b.** Generalize surface area formula of pyramids and cones (*SA = B + ½ Pl*).

8.G.12. Solve real-world and mathematical problems involving arc length, area of two-dimensional shapes including sectors, volume and surface area of three-dimensional objects including pyramids, cones and spheres.

UNIT 6: Probability

ESSENTIAL QUESTION	BIG IDEAS
How can properties of	Students will collect, organize and display data to find the probability of ev

probability be used to make educated decisions in real world situations?	and determine which outcomes are more likely to occur. Students understand the difference between experimental and theoretical probability and when to use each one. Students can conduct a probability experiment and analyze the results through a probability model to make educated decisions.

GUIDING QUESTIONS

Content and Process

- How can probability be expressed as a number between 0 and 1? **7.SP.5**
- What happens to the experimental probability as more trials are conducted? **7.SP.6**
- How can creating probability models be helpful to compare results with expected outcomes? **7.SP.**
- What factors cause differences between experimental and theoretical probabilities? **7.SP.7**
- How can probability models (uniform and non-uniform) be used to determine the likelihood of an event? **7.SP.7a, 7.SP.7b**
- How do lists, tables, tree diagrams, and simulations help determine the probability of compound events? **7.SP.8**
- How can a model (list, table, tree diagram) be used to represent the sample space of compound events and express the probability as a fraction? **7.SP.8a, 7.SP8b**
- How can a simulation be designed and used to generate frequencies for compound events? **7.SP.8**

Reflective

- When rolling two dice, what is the probability that the sum of the two numbers is 8?
- What are three ways to express the probability of rolling an odd number on a six-sided die?
- How can you explain to a friend why theoretical and experimental probabilities can differ?

FOCUS STANDARDS

Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

MP.8 Look for and express regularity in repeated reasoning.

Content Standards

7.SP.5. Express the probability of a chance event as a number between 0 and 1 that represents the likeliho of the event occurring. (Larger numbers indicate greater likelihood. A probability near 0 indicates an unlike event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.)

7.SP.6. Collect data from a chance process (probability experiment). Approximate the probability by observing its long-run relative frequency. Recognize that as the number of trials increase, the experiment probability approaches the theoretical probability. Conversely, predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be* *rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy

- **7.SP.7a.** Develop a uniform probability model by assigning equal probability to all outcomes, and u the model to determine probabilities of events. *For example, if a student is selected at random fro class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- **7.SP.7b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes fo the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation

- **7.SP.8a.** Know that, just as with simple events, the probability of a compound event is the fraction outcomes in the sample space for which the compound event occurs.
- **7.SP.8b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language *(e.g. "rolling double sixes")* identify the outcomes in the sample space which compose the event.
- **7.SP.8c.** Design and use a simulation to generate frequencies for compound events. *For example, random digits as a simulation tool to approximate the answer to the question: If 40% of donors hav type A blood, what is the probability that it will take at least 4 donors to find one with type A blood*

UNIT 7: Statistics and Sampling

ESSENTIAL QUESTION	BIG IDEAS
How can sampling and statistics be used to better understand a population?	Students will use data from a relevant sample to gain information and ma inferences about a larger population. Students will apply the appropriate measures of center and variability to r comparisons between sets of data.

GUIDING QUESTIONS

Content and Process Objectives

- How are statistics used to gain information about a population? **7.SP.1**
- What determines whether a sample is an accurate representation of a population? **7.SP.1a, 7.SP.1b**
- How can data from a sample be used to generate inferences about a population? **7.SP.2**
- How can multiple samples reveal the accuracy of a prediction or estimation? **7.SP.2**
- How are measures of center and variability used to make inferences and comparison between tw sets of data? **7.SP.3, 7.SP.4**

Reflective

- Explain multiple reasons why only polling the people at your lunch table about their favorite explc class would not be a representative sample of the entire school?
- What is a situation where it would be more appropriate to use the median instead of the mean? W
- If the average height of one sports team at your school is 10 cm taller than another team, what ca infer about the two teams?

FOCUS STANDARDS

Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

MP.6 Attend to precision.

Content Standards

7.SP.1. Use statistics to gain information about a population by examining a sample of the population;

- **7.SP.1a.** Know that generalizations about a population from a sample are valid only if the sample is representative of that population and generate a valid representative sample of a population.
- **7.SP.1b.** Identify if a particular random sample would be representative of a population and justify reasoning.

7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characte of interest. Generate multiple samples (or simulated samples) of the same size to informally gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled surv data. Gauge how far off the estimate or prediction might be.*

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (requires introduction of mean absolute deviation). *For example, the mean height of players on basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4. Use measures of center (mean, median and/or mode) and measures of variability (range, interqua range and/or mean absolute deviation) for numerical data from random samples to draw informal compar inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grac science book are generally longer than the words in a chapter of a fourth-grade science book. (NOTE: Students should not have to calculate mean absolute deviation but use it to interpret data).*